

## Bernoulli and Newton in Fluid Mechanics

Norman F. Smith

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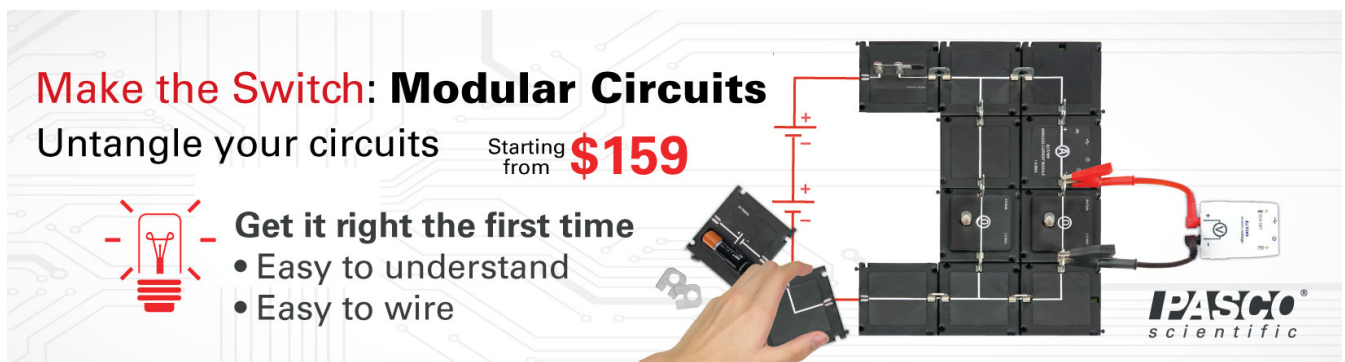
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# Bernoulli and Newton in Fluid Mechanics

Norman F. Smith

Millions of children in science classes are being asked to blow over curved pieces of paper and observe the fact that the paper “lifts,” or they are asked to blow between two suspended apples and to observe that the apples are drawn together. They are then asked to believe that Bernoulli’s theorem is responsible. They are also told that Bernoulli’s theorem is responsible for lift on the airplane wing and for the force that makes a spinning baseball travel in a curved path.

Unfortunately, the “dynamic lift” involved in each of these items is not properly explained by Bernoulli’s theorem. This theorem was never postulated to deal with dynamic lift but is only an expression for the law of conservation of energy inside an isentropic stream-tube of fluid. As such, it is concerned only with internal relationships inside the fluid. Dynamic lift must be examined as an external encounter between air and another object, an airfoil, for example. In such an examination, it becomes at once apparent that the law that must be used to describe this encounter is Newton’s third law covering action and reaction.

Because of this confusion about the basic principles of lift, considerable confusion exists at earlier levels in science education, and students are arriving in high school and college physics classes with an inadequate understanding of these concepts of fluid mechanics. To make matters worse, some of these errors, notably in material covering the curving baseball and the airplane wing, have become so popular that they are appearing in textbooks for high school and college physics and perhaps are being taught there.

From my contact with hundreds of teachers in aerospace workshops, I have found that the difficulty apparently begins with inadequate understanding of Bernoulli’s theorem. What does Bernoulli’s theorem really say and what is its basis? We must understand this clearly in order to know where and how it can be applied. Too often it is presented with only vague statements like “...when the velocity increases, the pressure decreases,” or “the pressure of moving air is less than that of still air,” with little or no explanation of what is behind it.

Bernoulli’s theorem is most easily understood as an example of the law of conservation of energy. Originally developed as an equation rather than a word statement, the theorem is often stated in fluid mechanics texts thus:

$$\frac{Wv^2}{2g} + \frac{Wp}{w} + Wz = \text{a constant,}$$

where  $p$  is static pressure,  $z$  is height above a reference plane,  $W$  is weight of fluid with volume  $V$ ,  $v$  is velocity,  $g$  is acceleration of gravity (gravitational field), and  $w$  is weight density.



**Norman F. Smith** is an engineer, pilot, and writer with long experience in aeronautics, fluid flows, and space science. After 18 years as an aeronautical research scientist with NACA/NASA, he spent 11 years in engineering management during projects Mercury, Gemini, and Apollo. At retirement (1970) he was Technical Assistant to the Director of Engineering and Development at NASA’s Manned Spacecraft Center at Houston. Mr. Smith now lectures on science and space and writes books and films for the educational world. His first book for general audiences was *Uphill to Mars, Downhill to Venus*, (Little, Brown, Boston, 1970); *Wings of Feathers, Wings of Flame* followed in April, 1972. Mr. Smith has served as technical authority and author of educational films on space for Time-Life and Coronet Instructional Films. (Route 1, Box 600, Alvin, Texas 77511.)

In metric units, this equation would be

$$\frac{1}{2} mv^2 + \bar{V}p + mgz = \text{a constant,}$$

where  $m$  is the mass of the fluid with volume  $V$ , and the other symbols have the same meaning as above. Translated into words, this equation says that in a fluid flow where no energy is being added or taken away, the kinetic energy represented by velocity (first term) plus the potential energy represented by pressure (second term) plus the gravitational potential energy represented by position (third term) equals constant total energy. The first and last terms will be immediately recognized as being dimensionally equal to energy; the second (which could be written  $pV$ ) may be a little less familiar, but is also a measure of stored (potential) energy.

In many fluid-mechanics problems, the third term is considered negligible and is omitted, while the density  $w$  is assumed constant. We can also divide through by  $W$  to make all terms per unit of weight. Making these simplifications leaves only two variables, velocity and static pressure:

$$v^2/2g + p/w = \text{a constant.}$$

Examined in this light, Bernoulli's theorem is clearly no anomaly or mystery but describes a simple interchange of kinetic and potential energy like that illustrated by a frictionless roller coaster. Bernoulli's theorem is best illustrated in devices such as the automobile carburetor, the venturi, and the jet pump. These devices all include a constricted section, or throat, where the flow is forced to increase its velocity (Fig. 1). The increased kinetic energy of the flow in the throat must come from the static pressure, because the total energy is constant. The resultant pressure decrease provides the suction needed in the carburetor throat, pump nozzle, or whatever.

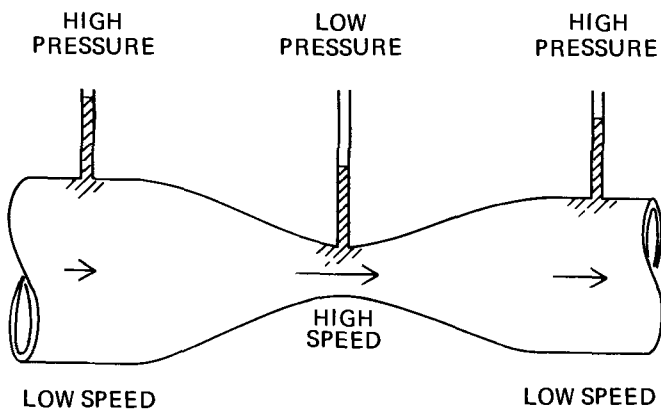


Fig. 1. Flow in a converging-diverging pipe is often used to illustrate Bernoulli's theorem.

In drawing an analogy between a roller coaster and a converging-diverging pipe, the constricted section corresponds to the low point on the roller-coaster track, where kinetic energy is greatest and potential energy is least, while the large section of the pipe corresponds to the high point of the track where the reverse is true.

Phenomena that involve an interchange of *velocity* and *pressure* in a constant-energy stream are valid applications of Bernoulli's theorem. But problems arise when Bernoulli's theorem is invoked to explain an encounter between two objects from which a *net force* results. The airplane wing and the curving baseball are in this category.

The airfoil of the airplane wing, according to the textbook explanation that is more or less standard in the United States, has a special shape with more curvature on top than on the bottom; consequently, the air must travel farther over the top surface than over the bottom surface (Fig. 2). Because the air must make the trip over top and bottom surfaces in the same elapsed time (or as some texts say, because air molecules parting at the leading edge must meet again at the trailing edge), the velocity over the top surface will be greater than over the bottom. According to Bernoulli's theorem, this velocity difference produces a pressure difference which is lift. (Some texts contain a hybrid explanation that describes lift as due to Bernoulli's theorem on top of the wing and Newton's law on the bottom).

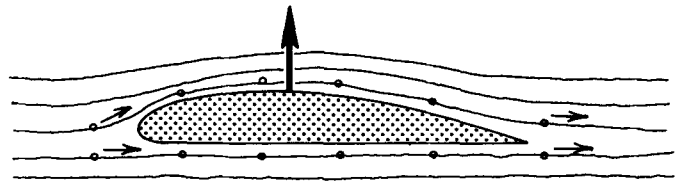


Fig. 2. A typical textbook diagram. Lift is produced, say the textbooks, when air traverses the top and bottom surface of a specially shaped airfoil in the same elapsed time.

Unfortunately, this explanation falls to earth on three counts. First, an airfoil need not have more curvature on its top than on its bottom. Airplanes can and do fly with perfectly symmetrical airfoils; that is, with airfoils that have the *same* curvature top and bottom. Second, even if a humped-up (cambered) shape is used, the claim that the air must traverse the curved top surface in the same time as it does the flat bottom surface (or that the molecules must meet again) is fictional. We can quote no physical law that tells us this. Third – and this is the most serious – the common textbook explanation, and the diagrams that accompany it, describe a force on the wing with no net disturbance to the airstream. This constitutes a violation of Newton's third law. Figure 2 is typical of diagrams found in most elementary and high school science textbooks.

In order to get at the true explanation for dynamic lift, we must look at the *encounter* between airfoil and air and seek out a physical law that describes this encounter. We can draw a parallel here with the explanation of static lift on a blimp or balloon. What holds a balloon aloft? Pressures on the balloon, of course – pressures that are greater on the bottom than on the top. These pressures are the *evidence* of lift, not its *cause*. The cause must be sought in the physics of the static encounter between air and balloon. It was Archimedes who first

discovered this principle; it now bears his name. Archimedes' principle is amenable to simple proofs that can be handled at the high school level – both experimental proof and the calculation of buoyant force on a simple submerged shape in terms of fluid depth and density. We can say that Archimedes' principle is firmly established behind static lift or buoyancy.

What supports an airplane aloft? Again, the answer is pressures on the airplane – pressures that are greater on the bottom than on the top of the lifting surfaces. And again, pressures are the *evidence* of lift, not its cause. The *cause* of the pressures must be sought in the dynamic encounter between air and airfoil. Archimedes can't help us much here, because buoyancy can only supply a negligible portion of the lift we need. Bernoulli can't help here either – his theorem just keeps saying that if there are pressure differences, there will be velocity differences; it tells us nothing about the principle of dynamics involved in the encounter.

Newton has given us the needed principle in his third law: if the air is to produce an upward force on the wing, the wing must produce a downward force on the air. Because under these circumstances air cannot sustain a force, it is deflected, or accelerated, downward.

Newton's second law gives us the means for quantifying the lift force:

$$F_{\text{lift}} = m\Delta v/\Delta t = \Delta(mv)/\Delta t .$$

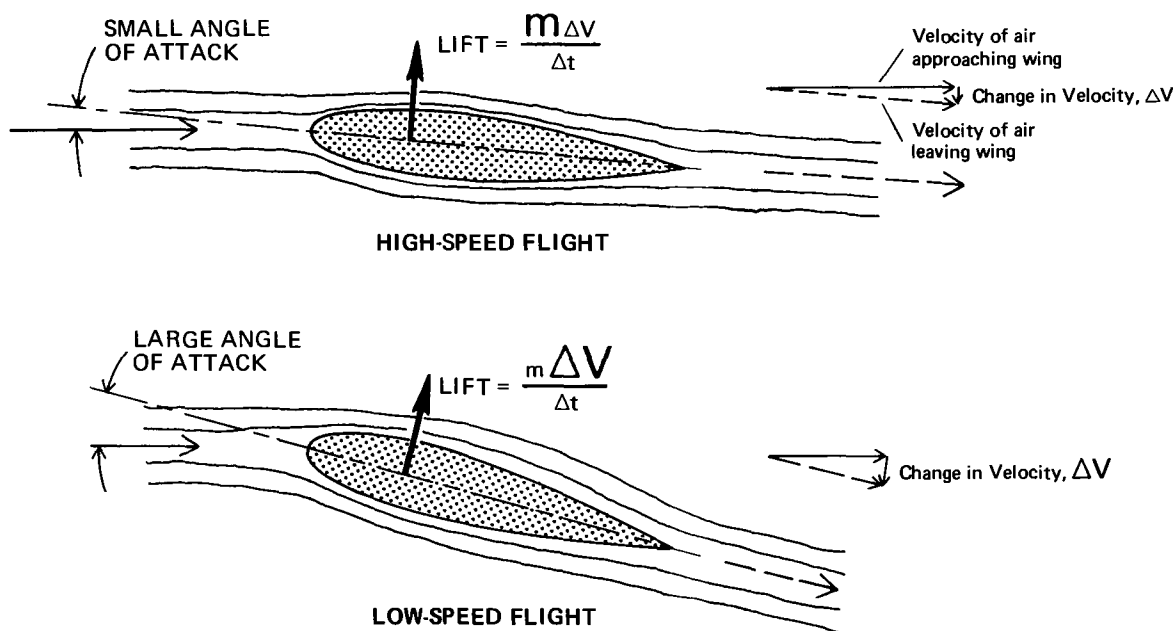
The lift force is equal to the time rate of change of momentum of the air. Because the lift is upward, the change of momentum must be downward. Figure 3 shows the correct physics of dynamic lift, and shows the vital relationship between angle of attack and speed known to every airplane pilot.

We cannot easily set up an experiment or a calculation of dynamic lift to prove this principle – as we can in the case of Archimedes' principle – because of difficulty in measuring or depicting in a "model" the size of the mass of air,  $m$ , that is deflected, and its average change of velocity,  $\Delta v$ . But we can (and do) demonstrate Newton's third law in the classroom in early science with such props as two children pushing on each other, and with more sophisticated laboratory devices in later science classes.

Because air is invisible, we cannot witness the action-reaction phenomena directly by watching an airplane, but we can get strong evidence from a helicopter. The pilot of a helicopter increases the speed of his rotor and increases the angle of attack of its blades until sufficient air is accelerated downward to give a reaction force equal to the weight of the machine. We can feel and see evidence of the downward blast if we stand nearby. Perhaps even more familiar to the millions who have watched Apollo spacecraft recoveries on television is the well-defined pattern of spray and waves created beneath a hovering helicopter by the downward-moving air.

The propeller also gives us direct qualitative evidence of the action-reaction requirement in the air blast generated when producing thrust. It gives us direct quantitative evidence, too, in the experimental use of momentum surveys to measure propeller thrust. NASA has published reports of many research programs in which propeller thrust was determined in both wind tunnels and in flight solely by measuring the momentum change through the propeller.

Strangely enough, although the wing is much simpler than the propeller, we cannot easily set up an experiment (or a calculation) to illustrate or prove the momentum



**Fig. 3.** The correct physics of airfoil lift: deflection of air downward, as required by Newton's third law. The same amount of lift can be produced by giving a large amount of air a small acceleration (top sketch) or a small amount of air a large acceleration (bottom sketch).

principle for dynamic lift of the wing. Such measurements are virtually precluded by the practical difficulties of measuring (or depicting in a model) the size of the air mass affected by the wing and the average acceleration given it. The best that we can do is to demonstrate and prove the principle of action and reaction and deduce its application to wing lift. The calculation of actual pressures on a wing should not be confused with the description of the physical principle of lift. Such mathematical procedures are part of the technology of aviation, not the basic physics.

For our purposes in teaching science and physics, then, we can say that Newton's law describes the dynamic encounter between air and *any* lift-producing body (kite, airplane, helicopter rotor, airplane control surface, etc.) and tells us what must happen in this encounter. *All* dynamic lift in a fluid (not just a part) requires the downward acceleration of air. This downwash-producing encounter is the *cause* of lift, while the pressures on the airfoil are the *effect*, or the result of this encounter. Attempts to explain the result (pressures) without looking for the cause have produced the "Bernoulli explanation" with its spurious logic, bad physics, and heritage of confusion.

The use of Bernoulli apparently began in this country some 30 years or more ago and has spread throughout school science books and popular literature to exclude virtually any mention of Newton and momentum. College-level aerodynamics textbooks generally do not use this approach, though it would not be surprising to find a few exceptions. Although the fundamentals are often obscured in the equations and the language of the technology, the explanation of lift in these texts is action/downwash equals reaction/lift. The McGraw-Hill *Encyclopedia of Science and Technology* says:

To obtain a force in a fluid the fluid must be accelerated – its momentum must be continuously changed. By Newton's Second Law the force is proportional to the rate at which momentum is imparted to the fluid .... The lift on a wing can also be related to the momentum changes in the flow field....Air must be pushed downward continuously to produce the lift.

*Encyclopedia Britannica* says of the helicopter:

Thrust (meaning lift) is produced by imparting a downward velocity to the mass of air flowing through the rotor. The lift is proportional to the change in momentum...

*The Encyclopedia of Physics* (Edited by Besancon, Reinhold Publishing Co.) says:

The overwhelmingly important law of low speed aerodynamics is that due to Newton....Thus a helicopter gets a lifting force by giving air a downward momentum. The wing of a flying airplane is always at an angle such that it deflects air downward. Birds fly by pushing air downward. Propellers and jet engines make a forward force (thrust) by giving air a rearward momentum.

It might be well to point out that the Bernoulli explanation is used mostly in the United States. The

British, for example, seem to use Newton's third law in textbooks and popular literature. In *Mastery of the Air* (Sutton Basic Books), the author says:

Lift depends essentially on the power of the wing to deflect air particles downwards, and the reason why a rapid forward motion is necessary in order to maintain a powerful sustaining force is that the wing must continually capture new air particles and drive them downwards. The more rapidly and efficiently it can do this, the stronger is the upward push or recoil.

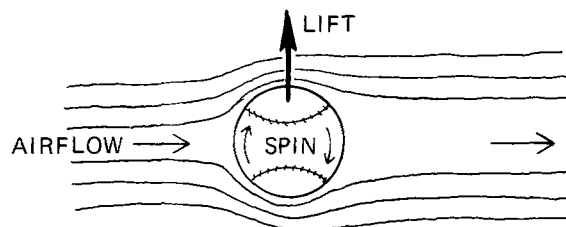


Fig. 4. A common textbook diagram. The spin, says the textbook, will cause lifting pressures upward on the ball. Such diagrams are incorrect. Because no net deflection of the air is shown, no lift can be claimed.

The use of Bernoulli's theorem to explain the lift on a spinning baseball (Fig. 4) is also incorrect for reasons similar to those outlined above. The ball cannot incur a "lift" force without producing an opposite thrust on the air. But the use of Bernoulli's theorem is incorrect on another count: the spinning baseball, which possesses kinetic energy in both its spinning and forward motion, exchanges energy with the surrounding air. On the top (Fig. 4) the ball adds energy to the airstream, through friction, to *increase* the air velocity locally, while on the bottom it takes energy from the air to *decrease* air velocity there. Because Bernoulli's theorem contains a stipulation that no energy will be added or taken away from the airstream, this theorem *cannot be applied* to a situation where energy interchange between ball and air is taking place. Despite its popularity, the Bernoulli explanation should be replaced by correct physics.

The explanation behind the lift generated by a rotating ball (or cylinder) is a little more complicated than that for the airplane wing, and involves some understanding of the interesting flow around spheres and cylinders. The flow over a stationary (nonspinning) sphere or cylinder is depicted in Fig. 5 and can be demonstrated by moving a large dowel through a tub or pan of water. The flow is smooth over the front face, but cannot follow the sharp curvature of the body to fill the space behind. Instead, it separates near the top and bottom of the circular contour and forms little vortices or whirls

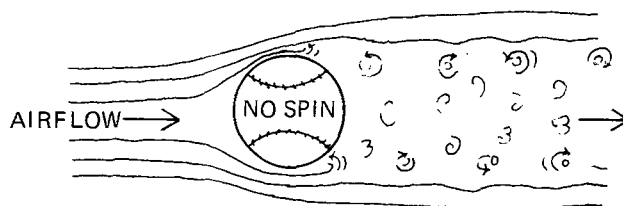


Fig. 5. When the ball is not spinning, the flow separates just behind the maximum diameter and flows straight back. There is no net deflection of the airstream and no lift.

that peel off alternately from top and bottom. The separated flow behind the circular shape is very rough, but symmetrical. No net lift is produced.

The point at which separation occurs is quite critical, and is easily affected by the conditions in the boundary layer. For example, adding roughness to the cylinder will often cause the boundary layer to change from laminar (layered) to turbulent (mixing). The greater mixing in a turbulent boundary layer gives the air close to the surface more energy; hence, it will travel farther around the circular (or spherical) shape before separating. As a result, a rough sphere may have less drag than a smooth one, which is one reason why golf balls are not built with a smooth cover, but instead are roughened with dimples.

Getting back to the spinning baseball, we can now make use of the very interchange of energy that invalidates the use of Bernoulli's theorem. The energy added and subtracted from the air flow over the ball affects the separation point drastically. With more energy, the air flowing over the top (Fig. 6) can follow the curved surface farther around the back side before separating. With less energy, the air flowing over the bottom is less able to follow the surface and separates farther forward on that side. The result is asymmetrical flow and a net deflection of air by the ball. It is this deflection, or acceleration, of air that produces the lift on a spinning baseball. It also explains the "slice" and "hook" that plague golfers. Asymmetrical flow (not Bernoulli's theorem) also explains lift on the ping-pong ball or beach ball that floats so mysteriously in the tilted vacuum-cleaner exhaust at the local department store.

A critical look at the material on Bernoulli's theorem in textbooks shows that Bernoulli's theorem is not taught

as well as it should be. It is no anomaly, no mystery, but a lucid example of the law of conservation of energy. Viewing velocity and pressure as manifestations of kinetic and potential energy, respectively, makes Bernoulli's theorem as easy to understand as the roller coaster.

A critical look at the applications of the laws of Bernoulli and Newton in science and physics suggests that improvement is needed here also. Bernoulli's theorem should be applied only to cases dealing with an interchange of velocity and pressure within a fluid under isentropic conditions. The carburetor, jet pump, and venturi are all valid applications of Bernoulli's theorem.

For explaining dynamic lift, the result of an encounter between a fluid and a lifting device, Newton's laws must be used. Consolidation of all dynamic forces produced in a fluid – propulsion, lift, control, etc. – under Newton's third law is not only correct physics but also makes the whole business far easier to teach and to learn.

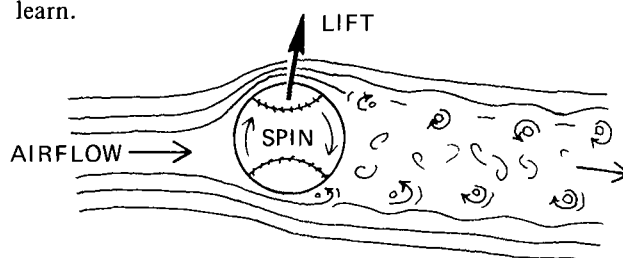


Fig. 6. When the ball is spinning, the spin imparts energy to the air on top of the ball, making the air able to follow the surface farther rearward before separating. Reverse is true on bottom of ball. The net deflection of the air-stream produces a lift reaction on the ball.

(LETTERS continued from page 427)

**MORE ON TRUE/FALSE**

During 15 years of college teaching I have avoided the use of the TRUE-FALSE technique of testing as being essentially invalid and useless as an evaluation device. Recently, I found myself in a position where it seemed that TRUE-FALSE testing might be used to advantage. By building on a suggestion by a colleague, and adding two of my own and one from a student, the following conditions were established for an all TRUE-FALSE test:

- (1) If the response F is chosen, the student is required to write a statement showing why the assertion is false.
- (2) Some of the assertions may appear in pairs, not necessarily adjacent, which state the same principle but in different words. The responses to both of the pair must be consistent (either both T or both F) in order to receive full credit for both.

- (3) Some of the assertions might be considered either T or F, depending upon viewpoint. A single response of T or F would receive half grade. Full credit only for responding TF and writing a statement expressing the two viewpoints.
- (4) An assertion can be true but incomplete. For example, an assertion requiring four hypotheses can be made including only two or three. Half credit for recognizing truth, full credit for adding the missing hypotheses.

I would be interested in hearing from persons: (a) who might add a few other conditions designed to increase the usefulness or validity of the TRUE-FALSE test or (b) who might give me a possible reference to work done by someone who has already investigated this possibility.

JOHN A. FYNN  
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**SURFACE SCREENING**

I took this picture (Fig.1) of the reflected image of the window screen in the distorted surface of a dish of water.

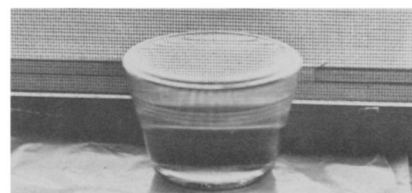


Fig. 1. The reflection of an undeformed window screen in a nonplanar water surface can be used to detect the shape of the surface. Try it with a floating greased sewing needle, a drop of oil, or a cork.

The many uses of this technique of observing the shape of the water surface will suggest themselves to your readers, I am sure, once the idea has been presented.

ELIZABETH A. WOOD  
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