INTRODUCTION TO METAMATERIALS

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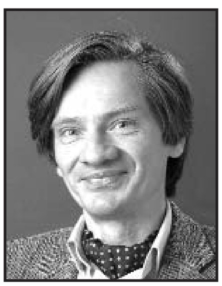

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Metamaterials (MMs) are artificial structures designed to have properties not available in nature [1]. They resemble natural crystals as they are build from periodically arranged (*e.g.*, square) unit cells, each with a side length of a. The unit cells are not made of physical atoms or molecules but, instead, contain small *metallic* resonators which interact with an external electromagnetic wave that has a wavelength λ. The manner in which the incident light wave interacts with these metallic "meta-atoms" of a metamaterial determines the medium's electromagnetic properties – which may, hence, be made to enter highly unusual regimes, such as one where the electric permittivity and the magnetic permeability become simultaneously (in the same frequency region) negative.

The response of a metamaterial to an incident electromagnetic wave can be classified by ascribing to it an effective (averaged over the volume of a unit cell) permittivity $\varepsilon_{eff} = \varepsilon_0\varepsilon_r$ and effective permeability $\mu_{eff} = \mu_0\mu_r$. In order to introduce such a description, one requires that the size of the artificial resonators characterized by a be much smaller than the wavelength λ, *i.e.* $a \ll \lambda$. As long as this criterion is fulfilled, one may normally assume that the response of the medium is local, *i.e.* that the values of ε_{eff} and μ_{eff} averaged over a given unit cell do not depend on the wavevector nor on the corresponding values of these parameters at neighboring unit cells. Hence, in such a medium, the effects of spatial dispersion may legitimately be ignored.

In this short review, we shall explain the fundamental characteristics of (negative-refractive-index) metamaterials, together with the manner in which their building blocks (meta-atoms) can be constructed. We shall also concisely outline an exemplary application that is enabled by such media, namely stopping of light. Finally, we conclude by outlining key challenges that need to be tackled before the deployment of metamaterials in such applications becomes more functional and efficient.

SUMMARY

The concept of man-made structures known as metamaterials is discussed along with their diverse applications, like stopping light and optical cloaking.

LEFT-HANDED MATERIALS

To describe the basic properties of metamaterials, let us recall Maxwell's equations

$$\nabla \times \mathbf{E} = -\mu_0\mu_r \frac{\partial \mathbf{H}}{\partial t} \text{ and } \nabla \times \mathbf{H} = \varepsilon_0\varepsilon_r \frac{\partial \mathbf{E}}{\partial t} \quad (1)$$

where the μ_r and ε_r are relative permeability and permittivity, respectively, and $\nabla = \left[\frac{\partial}{\partial_x}, \frac{\partial}{\partial_y}, \frac{\partial}{\partial_z}\right]$. From the above equations, one obtains the wave equation

$$\nabla^2 \mathbf{E} = -\varepsilon_0\mu_0\varepsilon_r\mu_r \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (2)$$

If losses are ignored and ε_r and μ_r are considered as real numbers, then one can observe that the wave equation is unchanged when we *simultaneously* change signs of ε_r and μ_r.

To understand why such materials are also called left-handed materials (LHM), let us assume a time-harmonic and plane-wave variation for fields in Maxwell's equations (1)

$$\mathbf{E}(x,y,z,t) = \mathbf{E}e^{i\omega t - i\mathbf{k}\cdot\mathbf{r}} \quad (3)$$

where we have introduced wavevector $\mathbf{k}$. Similar expression holds for $\mathbf{H}$. Then, Maxwell's equations take the form

$$\mathbf{k} \times \mathbf{E} = -\omega\mu_0\mu_r\mathbf{H} \quad (4)$$

$$\mathbf{k} \times \mathbf{H} = +\omega\varepsilon_0\varepsilon_r\mathbf{E} \quad (5)$$

From the above equations and definition of cross product, one can immediately see that for $\varepsilon_r > 0$ and $\mu_r > 0$ the vectors $\mathbf{E}$,$\mathbf{H}$ and $\mathbf{k}$ form a right-handed triplet of vectors, and if $\varepsilon_r < 0$ and $\mu_r < 0$ they form a left-handed system (see Fig. 1) – from where the designation of these media as left-handed arises.

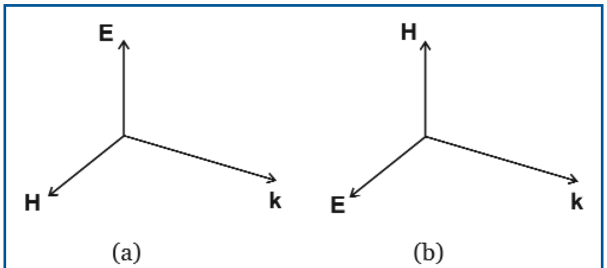

Fig. 1 (a) Right-hand orientation of vectors **E**, **H**, **k** for the case when $\varepsilon_r > 0$, $\mu_r > 0$. (b) Left-hand orientation of vectors **E**, **H**, **k** for the case when $\varepsilon_r < 0$, $\mu_r < 0$.

An important difference between regular dielectrics and left-handed metamaterials can be realized when one considers propagation of a ray through the boundary between left-handed and right-handed media, as shown in Fig. 2. Here, 1 is the incident ray, 2 is the reflected ray, 3 is the refracted ray when second medium is the right-handed, and 4 is the refracted ray when the second medium is left-handed.

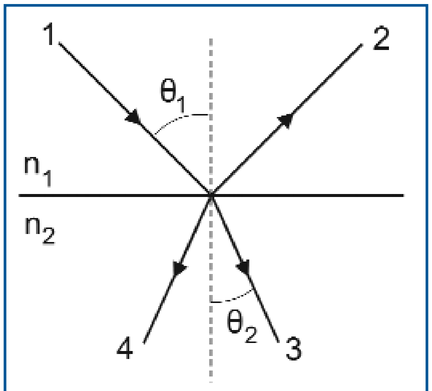

Fig. 2 Reflection and refraction at the interface of two media with $n_1 > 0$ and $n_2 > 0$ (ray 3) or $n_2 < 0$ (ray 4).

Light crossing the interface at non-normal incidence undergoes refraction, that is a change in its direction of propagation. The angle of refraction depends on the absolute value of the refractive index of the medium and it is described by Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{6}$$

By matching the field components at the dielectric interfaces, one may readily verify that in the case where the second medium is double-negative ($\varepsilon_r < 0$, $\mu_r < 0$), the refraction of light occurs on the same side of the normal as the incident beam (see Fig. 2). Thus, we see that the double-negative medium (that is medium with $\varepsilon_r < 0$ and $\mu_r < 0$) behaves as a medium exhibiting a negative (effective) refractive index.

ELEMENTARY CELL OF METAMATERIAL FORMED BY SRR AND THIN WIRE

In recent years several elements of various structures have been considered as building blocks (unit cells) of metamaterials [2-4]. Provided that the dimensions of such unit cells are much smaller than the wavelength, one can determine the effective relative magnetic permeability μ_r, and electric permittivity ε_r, by proper averaging techniques. In the case where both of these parameters are negative, the correct value that should be attributed to the effective medium's refractive index is given by

$$n_{eff} = -\sqrt{\mu_r \varepsilon_r} \tag{7}$$

The negativity of the real part of a medium's refractive index in the case where the real parts of the permittivity and permeability are negative is a general result, valid for all kinds of passive and active media [5]. However, caution needs to be exercised in the cases where either the permittivity or the permeability of the metamaterial is active [5].

Let us now discuss how one can create media having negative permittivity and negative permeability.

Metamaterials with negative effective permittivity in the microwave regime

It is well-known that metals at optical frequencies are characterized by an electric permittivity that varies with frequency according to the following, so called Drude, relation

$$\varepsilon(\omega) = \varepsilon_0 \left[1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \right] \tag{8}$$

where $\omega_p^2 = \frac{Ne^2}{m\varepsilon_0}$ is the plasma frequency, *i.e.* the frequency with which the collection of free electrons (plasma) oscillates in the presence of an external driving field, N, e and m being, respectively, the electronic density, charge and mass, and γ is the rate with which the amplitude of the plasma oscillation decreases. One can directly infer from Eq.(8) that, *e.g.*, when $\gamma = 0$ and $\omega < \omega_p$ it is $\varepsilon < 0$, *i.e.* the medium is characterized by a negative electric permittivity. Typical values for ω_p are in the ultraviolet regime, while for γ a typical value (*e.g.*, for copper) is $\gamma \approx 4 \times 10^{13}$ rad/s. Unfortunately, for all frequencies $\omega < \omega_p$ for which $\varepsilon < 0$, it is also $\omega \ll \gamma$, *i.e.* the dominant term in Eq.(8) is the imaginary part of the plasma electric permittivity, which is associated with losses (light absorption).

A method for overcoming this limitation was first proposed and analyzed in detail by Pendry *et al.* [6], based on the observation that the plasma frequency depends critically on the density and mass of the collective electronic motion. They considered the structure illustrated in Fig. 3, wherein thin metallic wires (infinite in the vertical direction, z) of radius r are periodically arranged on a horizontal plane (xy). The unit cell of the periodic structure is a square whose sides have length equal to a. If an electric field $\mathbf{E} = E_0 e^{-i(\omega t - kz)}\mathbf{z}$ is incident on the structure, then the (free) electrons inside the wires will be forced to move in the direction of the incident field. If the wavelength of the incident field is considerably larger compared to the side length of the unit cell, $\lambda \gg a$, then the whole structure will appear (to the incident electromagnetic field) as an effective medium whose electrons (confined in the wires) move in the $+\mathbf{z}$ direction. The crucial observation here is that, since the electrons are confined to move only inside the thin wires, the effective electron density of the *whole* structure (effective medium) is $N_{eff} = N\frac{\pi r^2}{a^2}$, with N being the

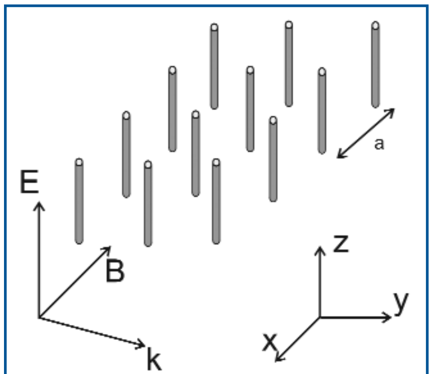

Fig. 3 Schematic illustration of a period arrangement of infinitely long thin wires, used in the creation of an effective plasma medium at microwave frequencies.

electron density inside each wire. Thus, for sufficiently thin wires the effective electron density, N_{eff}, of the engineered medium can become much smaller compared to N, thereby substantially decreasing the effective plasma frequency, ω_p, of the engineered medium. For instance, for a wire radius $r = 1\mu m$ and wire spacing $a = 5$ mm, we find that $N_{eff} \approx 1.3 \times 10^{-7}N$, *i.e.* the effective electronic density of the new medium is reduced by seven orders of magnitude compared to that of the free electron gas inside an isolated wire.

Moreover, it also turns out that the effective mass, m_{eff}, of the electrons moving inside the engineered medium is considerably larger compared to the free electron mass, m.

One can show that the effective mass m_{eff} of a moving electron inside our effective medium is $m_{eff} = 0.5 \times \mu_0 N e^2 r^2 ln(a/r)$. Thus, for copper wires of radius $r = 1\mu m$, being separated by $a = 5$mm, we obtain: $m_{eff} \approx 1.3 \times 10^4 m$, *i.e.* the effective mass of an electron in our engineered medium is increased by more than four orders of magnitude. This, combined with the fact that the effective electron density is reduced by approximately seven orders of magnitude, leads to an effective plasma frequency that is in the microwave regime

$$\omega_p^2 = \frac{N_{eff} e^2}{m_{eff}\varepsilon_0} = 5.1 \times 10^{10}\,[rad/s]^2 \\ \rightarrow f_p = \omega_p / 2\pi = 8.2\ GHz \tag{9}$$

It should be noted that, based on Eq. (9), the calculated wavelength, $\lambda_p = c/f_p$, which corresponds to our medium's effective plasma frequency turns out to be considerably larger compared to the periodicity of the structure ($\lambda_p \approx 7a$), justifying the description of the periodic structure as an effective medium. Therefore, with the herein presented methodology, we are indeed able to construct an engineered medium that can exhibit a *negative* electric permittivity in the *microwave* regime (with reasonably low losses and high field penetration inside the structure), thereby mimicking the interaction of light with real metals in the optical regime.

Metamaterials with negative effective permeability in the microwave regime

In the previous section we examined how we can construct a metamaterial exhibiting negative electric permittivity (ε_r) in the microwave regime. However, harnessing the remarkable properties of left-handed metamaterials also requires a design strategy for obtaining negative (effective) magnetic permeability (μ_r) at the same frequency region. Unfortunately, with the exception of some magnetic gyrotropic materials, media exhibiting negative μ_r do not occur naturally and should, thus, be built in the lab.

In this section we shall see how one can construct such magnetic metamaterials in the microwave regime using entirely non-magnetic structured metallic elements [7], which act as the magnetic 'molecules' of the engineered medium.

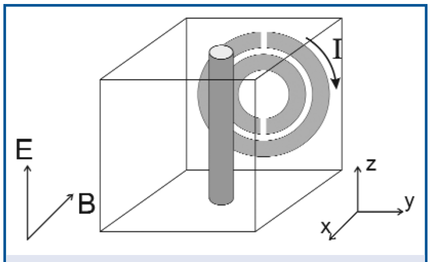

Fig. 4 Elementary cell of metamaterial formed by SRR and thin wire.

To this end, consider a three-dimensional periodic repetition of the external (larger) ring, shown in Fig. 4. The radius of the ring is r, and the whole arrangement is assumed to be immersed in air. This 'split ring resonator' (SRR) is equivalent to a simple RLC circuit, R being the resistance of the metallic ring, L its inductance and C (primarily) the capacitance between its unconnected ends. The rings residing on a given $x = x_i$ plane have the same axis (*i.e.*, they are 'concentric') with the corresponding rings on the x-planes below and above them. The side of the square unit cell on an yz plane is equal to a.

Assuming that a magnetic field $\mathbf{H} = H_0 e^{i(\omega t - k \cdot r)}\mathbf{x}$ is incident on the structure, the induced (electromotive) source is $U = i\omega\mu_0\pi r^2 H_0$, generating an electric current I that circulates in each ring (see Fig. 4). If the rings sitting on successive x-planes are close together ('solenoid' approximation) there will be negligible 'loss' of magnetic flux between the rings in each column, and therefore the magnetic flux will be $\Phi = \mu_0\pi r^2 I/l$, l being the x-distance between corresponding SRRs lying on successive yz planes. Accordingly, the inductance L (in Henry) of each SRR will be: $L = \Phi/I = \mu_0\pi r^2/l$. One may further assume that the depolarizing magnetic flux lines generated by all rings are uniformly spread on a given yz plane, which results in a mutual inductance between two SRRs given simply by: $M = (\pi r^2/a^2)L = FL$, F being the fractional volume within a unit cell occupied by an SRR. We may now apply Ohm's second law across a closed SRR 'circuit' to obtain: $U = [R + i/(\omega C) - i\omega L + i\omega M]I$, where $R = 2\pi r\sigma$ is the (ohmic) resistance of each ring, σ being the resistance per unit length. Thus, the induced magnetic dipole moment per unit volume, M_d, will be $M_d = I(\pi r^2)/(a^2 l)$, with the current I inferred from above equation to be

$$I = -\frac{H_0 l}{(1-F) - 1/(\omega^2 LC) + iR/(\omega L)} \tag{10}$$

As a result, the (relative) effective magnetic permeability associated with this medium will be (in the direction, x, that the incident magnetic field is polarised)

$$\mu_r = \frac{B/\mu_0}{B/\mu_0 - M_d} = 1 - \frac{F}{1 - 1/(\omega^2 LC) + iR/(\omega L)} \tag{11}$$

From Eq.(11) we can see that μ assumes negative values in the range: $1/\sqrt{LC} < \omega_p < 1/\sqrt{LC(1-F)}$, where $\omega_{m0} = 1/\sqrt{LC}$ is the resonance frequency of the Lorentzian variation of the medium's magnetic permeability, and $\omega_{mp} = 1/\sqrt{LC(1-F)}$ is the corresponding plasma frequency (where $\mathrm{Re}\{\mu\} = 0$). Crucially, we note that the resonant wavelength (λ_{m0}) of the structure depends entirely on the rings' effective inductance (L) and capacitance (C), and can therefore be made considerably larger that the periodicity (a) of the structure, thereby fully justifying its description as an effective medium. Had we placed the SRRs on the other two planes (xy and xz) we would have, similarly, obtained negative effective permeabilities in the other two directions, y and z, as well, and the variation with frequency of these permeabilities would have been given by an expression similar to Eq. (11). Thus, with the present methodology we are able to construct a three-dimensional, isotropic metamaterial exhibiting negative effective permeability in a specified frequency region.

We finish this Section by showing in Fig. 5 plots of real parts of effective permittivity (ε_{eff}) and permeability (μ_{eff}).

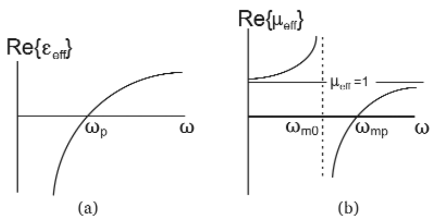

Fig. 5 (a) Permittivity of wire medium demonstrating plasma-like frequency dependent permittivity and (b) Frequency response of effective permeability.

POSSIBLE APPLICATIONS

Having materials with such unusual properties, there is no doubt that the so-enabled applications will also be unusual. We do not attempt to review all of them, we just concentrate on one interesting possibility, namely how to stop light. In passing, however, we would like to highlight some key possible applications that have emerged, such as 'perfect' lenses where one can beat the fundamental limit established by the law of diffraction [8], and the possibility of creating an optical analogue of a black hole [9,10].

The electromagnetic black hole was build recently [11]. We also mention the creation of an 'invisibility' cloak with the use of metamaterials [12,13].

For decades scientists maintained that optical data cannot be stored statically and must be processed and switched on the fly. The reason for this conclusion was that stopping and storing an optical signal by dramatically reducing the speed of light itself was thought to be infeasible.

However, recently a method has been proposed that can allow for a true stopping of light [14]. Indeed, there is currently a considerable interest in metamaterial waveguide structures capable of dramatically slowing down or, even, completely stopping light. The deceleration of light in this structures is associated with a negative Goos-Hänchen (G-H) phase shift – a lateral displacement of light ray when it is totally reflected at the interface of two different dielectric media as illustrated in Figs. 6-8.

To more precisely understand the manner in which light is decelerated in this structure, let us imagine a ray of light propagating in a zigzag fashion along a waveguide with a negative-index ('left-handed') core. The ray experiences negative Goos-Hänchen lateral displacements each time it strikes the interfaces of the core with the positive-index ('right-handed') claddings, see Fig. 7. Accordingly, the cross points of the incident and reflected rays will sit inside the left-handed core and the effective thickness of the guide will be smaller than its natural thickness. It is reasonable to expect that by gradually reducing the core physical thickness, the effective thickness of the guide will eventually vanish. Obviously, beyond that point the ray will not be able to propagate further down, and will effectively be trapped inside the negative index metamaterial (NIM) heterostructure, see Fig. 8.

The authors of [14] envisioned slowing and stopping a light pulse by varying the thickness of the waveguide core to the point where the cycle-averaged power flow in the core and the cladding become comparable. At the degeneracy point, where the magnitudes of these powers become equal, the total time-averaged power flow directed along the central axis of the core vanishes. At this point the group (or energy) velocity goes to zero and the path of the light ray forms a double light cone ('optical clepsydra') where the negative GH lateral shift experienced by the ray is equal to its positive lateral displacement as it travels across the core. Adiabatically reducing the thickness of the NIM core layer may, thus, in principle, enable complete trapping of a range of light rays, each corresponding to a different frequency contained within a guided wavepacket.

This ability of metamaterial-based heterostructures to dramatically decelerate or even *completely stop* [15] light under realistic experimental conditions, has recently led to a series of experimental works [16-17] that have reported an observation of, so called, 'trapped rainbow' light-stopping in metamaterial waveguides – to our knowledge, the first experimental works to provide a telltale spectroscopic fingerprint of 'true' light-stopping in solid-state structures.

LOSSES IN NIM

The full exploitation of optical metamaterials suffers from the existence of relatively high dissipative losses, which at present are orders of magnitude too large for practical applications, and are considered as an important factor that limits practical appli-

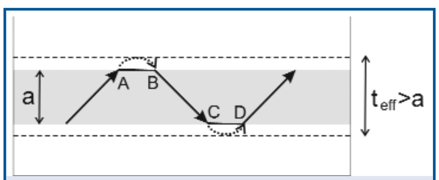

Fig. 6 Classical G-H shift in regular dielectrics.

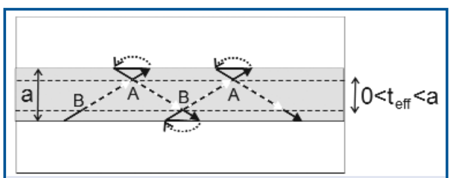

Fig. 7 G-H shift in NIM forming central layer.

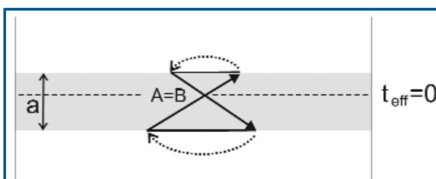

Fig.8 G-H shift in NIM at critical thickness.

cations of metamaterials. Whether losses can be overcome in realistic metamaterials has, some time ago, been the subject of a controversy [18]. However, it has recently been shown theoretically by Webb and Thylen [19], and Kinsler and McCall [20,21], and Skaar [22] that it is indeed possible to completely eliminate losses in photonic metamaterials by using active gain inclusions into the metamaterial structure.

Furthermore, several recent computational [23-25] and experimental [26] works have conclusively demonstrated that optical losses can be fully overcome in realistic negative-refractive-index metamaterials. The specific loss-free design considered in [23] and [26] consisted of two metallic films perforated with small rectangular holes ('fishnets'), and with an active medium (laser dyes) spacer between the two films. This configuration was shown to result in plasmonically-enhanced electric fields between the two closely spaced metallic films, which resulted in enhanced gain-harnessing by an incident optical field (since stimulated emission is proportional to $|E|^2$). Using ultrashort (ps and fs) pulses, the investigators of [23] and [26] were able to show that the detrimental effects of amplified spontaneous emission (ASE) noise could be overcome, since the overall amplification of light through the negative-index 'fishnet' metamaterial was occurring on very short timescales – much shorter that the ns timescale associated with ASE noise in the gain medium. These important developments have helped to establish that NRI metamaterials can, indeed, when judiciously designed, be made loss-free and even amplifying – thereby opening the road to a wealth of exciting and useful applications [27].

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